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The Research and Science Behind Aha!Math™

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I. INTRODUCTION

Technology-supported Learning: Are We There Yet?

Classroom instruction today is typically embedded with long-standing practices and a sprinkling of new programs and learning tools. Classrooms are busy places filled with students, books, paper and pencils, decorative environments, the occasional computer, and, of course, a teacher. While the comfort and familiarity of the traditional classroom and its instructional resources reassure even the most discriminating eye, most know that learning could be improved. Furthermore, most educators, wanting the best for their students, suspect that technology applications, well designed and supported, could play a greater role in the advancement of education. In short, our educational past has met what was once considered the future, and the present now represents a new paradigm of technology-based teaching and learning.

Philosopher of science Thomas Kuhn defined a paradigm as “an entire constellation of beliefs, values and techniques, and so on, shared by the members of a given community.” Paradigms represent a pattern or a model of a system; certainly elementary education in the United States is an exemplar of a paradigm. As educators acknowledge the K-5 paradigm that has been created over the past 100 years, it should be noted that the classroom setting has been slow to change due to two primary factors. First, educators find it easier to embrace classroom changes when strong evidence of success connected to these changes exists. Unfortunately, for some time that evidence of success has been limited and unconvincing. And, second, practicality must be taken into account; is there enough resource (time, money, expertise, etc.) to implement the change? While it is good to “think outside the box,” the requirements for accountability are unforgiving. Therefore, paradigm shifts in education are traditionally not born solely of new ideas or hasty decisions, but rather of substantial, time-proven methods or practices that can be feasibly achieved in the context of schooling.

For 20 years the introduction of technology-based resources in the classroom has been touted, perhaps prematurely, as a paradigm shift in education. Since the first applications of computers in the classroom we have learned an enormous amount about the interactions and relationships of teachers, students, computers, software, and learning. We are grateful to those who were quick to embrace technology in the classroom, as they proved to be the Petri dish for many others. And, while classroom environments were being tested for technology-based learning applications, a whole new world of technology was being explored via home use of game-based software, the Internet, and Web-based programs.

We have now reached an important juncture in our journey to improve learning with technology-based resources. We now have credible research, case studies, and a multitude of tested contexts that can guide the course for effective uses of technology to enhance teaching and learning. We have scholars and scientists who have spent years studying the effects of technology-driven opportunities to learn. We have collected teacher and parent opinion about technology and learning. We have test scores and achievement records that convey the effectiveness of technology-enhanced curricula. We have evidence of student behaviors, preferences, and performance. And, we at Learning.com have the expertise from developing and supporting two award-winning Web-enhanced solutions for students in grades K-8. Learning.com has made it our business to dedicate the time needed to extrapolate the best research findings, listen to educators, identify student needs, and to continue to build state-of-the-art enhanced instructional support for all students. It is high time to provide educators with the evidence they require and the practical solutions they seek to make this paradigm shift to technology-enhanced learning into a common practice.

This white paper introduces Learning.com's newest learning solution, Aha!Math, a Web-delivered, K-5 supplemental math program designed in a way that captures our best understanding of how students learn math and how they maximize the benefits of technology-enhanced learning opportunities. It addresses five prominent areas of concern that challenge educators as they strive to deliver effective math instruction.

The format of this white paper has been adapted for educator use. It describes the K-5 learning problems, connected solutions, and references to research findings and recognized works.

II. K-5 MATHEMATICS: FIVE AREAS OF CONCERN

As our nation institutionalized academic standards and accountability as the core mission in K-12 education, instructional practices began to change to reflect the priorities of the No Child Left Behind (NCLB) act. Educators are now clearly focused on what students need to learn, have implemented more targeted teaching methods, and have access to new ways to monitor student progress and mastery; we are seeing student achievement on the rise. It is an exciting time to be involved in education, and to accept the challenge of ensuring all students learn a rigorous curriculum. However, the real question becomes, "How do we continue to achieve the ever-increasing expectations?" Progress to date has been commendable, but far more is needed for all students to reach proficiency in mathematics by the year NCLB-mandated 2014.

1. Area of Concern: How to Achieve Effective Mathematics Instruction in a Standards-based Environment:

With the advent of national and state mathematics standards, school districts realized the need to ensure student access to the curriculum they were accountable to learn. Teachers turned their attention to grade-level standards – *when* the standards would be taught – and what materials could be used to teach the standards. Mathematics instruction became highly structured, organized for timely delivery and monitoring. Problems began to emerge. Textbooks included many more standards than the grade-level standards, creating alignment and instructional unit concerns. Checklists of standards frustrated teachers as they attempted to teach standard-by-standard, highly concerned about covering all of the standards and painfully aware that not all standards are equal. Clearly, the emphasis on mathematics standards for instruction has yet to yield the mastery of mathematical concepts and relationships intended by state standards.

2. Area of Concern: Adequate Time to Teach and Learn Mathematics

The current renewed focus on literacy has led to more instructional time devoted to reading instruction, at the cost of other areas, including mathematics. Ensuring that students could read promised to open the doors to all other subject areas, enhancing both the quantity and quality of what a student could learn. While mathematics has always been considered by educators to be a cornerstone of learning, we now recognize that even mathematics instruction time was limited to afford the time for literacy instruction. In most cases, student achievement in mathematics has remained flat. Experienced elementary math teachers frequently share concerns regarding the quality of math instruction, and the ability to fully develop a student's conceptual understanding and computational fluency of mathematics. Experts are increasingly expressing worries that there is a need to increase learning time and mathematics lessons that go beyond drill and practice to enable students to succeed in math. Much like literacy, mathematics will open the doors to and increase learning in other subjects like science and economics, for example.

3. Area of Concern: The Lack of Research-based Methods for Teaching and Learning Mathematics

Enhancing mathematics instruction and learning through technology is not a new idea. However, most technology solutions designed to support math instruction replicate the same lessons contained in basal texts. Many add a scoring mechanism, affording students immediate feedback regarding their performance. And, generally, the design of most mathematics software promulgates students working alone on a computer, isolated from all others. What is missing is the incorporation of the research behind how best to use technology to teach and learn mathematics in ways that support students to not only practice and master their math skills, but also to support opportunities for coaching, deeper exploration and inquiry, and collaboration.

4. Area of Concern: Students' Lack of Motivation to Learn Mathematics

Engaging an elementary student in math instruction, asking him or her to participate in independent practice, and relying on student-initiated improvement with limited feedback is challenging at best. While some students are independent learners, more often than not teachers struggle with students who resist classroom lessons, independent practice, and homework. Educators often describe this difference in terms of student motivation.

Technology has the capacity to engage students in learning activities. Recent research illustrates not only the increased interest in learning when students use computers, but also an increase in the length of time students remain engaged. However, without a good foundation of understanding student motivation, principles of learning, and performance feedback, technology can only amplify classroom frustrations.

5. Area of Concern: The Need to Improve Mathematics Teaching and Coaching

The link between quality teaching and improved student achievement has never been more clear (Haycock, 1998; Ma, 1999; National Commission on Teaching & America's Future (U.S.), 1996). To teach for understanding, teachers must thoroughly understand the subject matter so they can present it in a challenging, clear, and concise manner (National Commission on Teaching & America's Future, U.S., 1996). It means having a deep understanding of the subject matter so one can represent the discipline in multiple ways: "[clothed] in activities and emotion, in metaphors and exercises, and in examples and demonstrations, so that it can be grasped by students," (Shulman, 1987, p. 13). It means finding opportunities to reflect and adapt one's practice so sound decisions about student learning are made. Teaching for understanding acknowledges that learning varies by student and grade level as well as by content and curricular goals (Darling-Hammond & Sykes, 1999). "The real mathematical thinking going on in a classroom, in fact, depends heavily on the teacher's understanding of mathematics" (Ma, 1999, p. 153). It is the teacher who makes the final decisions about what to teach, when to teach, and how to teach (Fennema & Romberg, 1999; Leinhardt & Smith, 1985; National Council of Teachers of Mathematics, 2000; Steinberg et al., 1985). Therefore, changing a teacher's understanding is central to shifting the way in which teaching and learning occur in schools.

Poor undergraduate preparation of elementary math teachers, combined with limited resources for professional development and a national emphasis on literacy, has created a sizable problem: Teachers lack the knowledge needed to teach for deeper mathematical understanding. Added to these problems is the lack of experienced coaches for students; parents often do not have the time and/or ability to support their child's efforts to continue his or her learning at home, for example.

III. RESEARCH-BASED SOLUTIONS BUILD AHA!MATH

To achieve depth, understanding, and proficiency in mathematics, students must master the core concepts in math that lay the foundation for further learning.

Teaching and learning mathematics is a complex task. To better understand how children learn to compute, think critically, and problem solve, one must first understand the difference in “learning by following” and “learning by understanding.” Mathematics proficiency is built through a series of steps and stages that connect parts of what students learn to a greater whole. Students who “learn by following” can replicate the teacher’s work or the book’s examples; however, when faced with the next step or stage of learning they become confused or frustrated because they cannot explain the “why” of what they learned. In contrast, students who “learn by understanding” are ready for the next steps or applications because they know why, have tested the hypothesis, applied their learning in a variety of contexts, and are confident about moving forward with their learning. The elementary years of math instruction are especially important for building not merely the skills, but also the understanding needed for further learning. It is our premise that students need greater opportunities to solidify foundational math skills and knowledge through a deeper, richer application of their learning.

Aha!Math is designed to supplement instruction for a basal math curriculum. It is not a drill-and-practice, frequent-assessment, or multiple-choice driven program. Aha!Math provides the classroom teacher and his/her students with an opportunity to develop greater math proficiency through Web-based visuals, exercises, and activities – all of which have been designed for individual, small group, or whole class interaction.

Learning.com created Aha!Math using a matrix of scientific, research-based findings based on the foundations of teaching and learning mathematics. Our science of “teaching for learning” model draws upon four significant fields of research and experience from the most prominent representative professional associations:

1. Educational Research: National Council of Teachers of Mathematics (NCTM), Association for Supervision and Curriculum Development (ASCD), the American Educational Research Association (AERA), International Society for Technology in Education (ISTE), and the International Reading Association (IRA).

2. Psychological Research: The American Psychological Association and the Society for Personality and Social Psychology (SPSP).
3. Cognitive Science Research: Cognitive Science Society, the International Society of the Learning Sciences (ISLS), and the International Social Cognition Network.
4. Neuroscience Research: The Society for Neuroscience and the Society for Cognitive Neuroscience.

What Students Learn

Student achievement in mathematics is dependent on learners having a clear understanding of the ***learning targets***. Teachers rely on standards as targets to guide instruction. However, moving from state to state, there is little clarity surrounding how standards are defined and what students are expected to do. For example, New York requires first graders to be proficient in 30 process indicators and 56 content performance indicators. Indiana, on the other hand, asks first graders to be proficient in five process indicators and 35 content indicators. Several states have multiple standards documents, causing confusion as to expectations, while a few states still have learning expectations that exist for a grade span even though assessments differ at each grade level.

Each state develops its own standards and their implementation into what is taught often lies in the hands of state, local, and school officials. Learning outcomes expectations differ not only state by state but also as to the grade in which a concept should be learned. While this autonomy is expected in the United State, it hinders the development of instructional materials that are in depth, rigorous, and focused.

With the passage of NCLB, many states have rewritten their math standards. In a recent study of state standards, 31 of the 50 states plus the Department of Defense Education Agency (DoDEA) and the District of Columbia (DC) have written new or draft standards since 2003. As states begin to understand better the impact standards have on instruction, assessment, instructional materials, and student learning, they are looking to national leadership to assist in the development of new standards. While 81 percent of state participants say that math standards were developed to provide direction to teachers, they were greatly influenced by the National Council of Teachers of Mathematics (NCTM) standards that focus on a comprehensive mathematics continuum of learning. This is evident in the use of the five content standards and the five process standards for most states in grade K-8.

Aha!Math concentrates on a ***continuum of learning targets*** representative of the NCTM ***Curriculum Focal Points***. Developed to help math educators differentiate the most important math concepts and skills from laundry lists of standards, the Curriculum Focal Points represent, according to NCTM, “the mathematical understanding, knowledge, and skills that students should acquire.” These key curriculum focal points also embody learning goals and objectives and student expectations in a more thematic, unit-driven approach. The design of Aha!Math’s content is, therefore, easy to align to standards and provides teachers and students with greater opportunities to explore mathematical concepts in the context of related content and connected applications.

How Students Learn

Using the NCTM Curriculum Focal Points as the foundation, Aha!Math then builds on its ***teaching for learning model*** to incorporate six critical areas of learning. The following construct guides both the teaching and learning process in Aha!Math:

1. Content link to cognitive levels
2. Invitation to inquiry
3. Modeling and meaning-making
4. Learning activities
5. Feedback
6. Learning assessment

Learning mathematics is more than knowing a formula or remembering a procedure or solving a problem. The National Research Council defined mathematical proficiency, or helping all students to successfully learn mathematics, as having five strands that are interwoven and interdependent. They are ***understanding, computing, applying, reasoning, and engaging***.

Hiebert et.al. writes “that we understand something if we see how it is related or connected to other things we know.” The degree to which one understands rests on the connections or relationships and the richness of these relationships. ***Understanding*** a topic provides the foundation for remembering or reconstructing mathematical facts or methods. With this definition, understanding is also known as conceptual knowledge. Instruction that builds conceptual knowledge helps students link old knowledge with new knowledge. This means providing time for reflection and communication. As an example, students conceptually understand multiplication of two numbers when they have made the connections that two repeated four times is the same amount as four repeated two times.

Computing refers to student fluency in carrying out mathematical procedures – effectively, accurately, and with a degree of flexibility. Heibert calls this *procedural knowledge* and defines it in two parts: "formal language, or symbol representations system, of math [and] … algorithms, or rules, for completing mathematical tasks." Students must learn both procedures and when to use them.

The strand of ***applying*** refers to "being able to formulate problems mathematically" and to use one's ability to connect procedures and concepts to solve problems. It is critical to know what to do to solve a problem as well as what does not apply to that particular problem. Both understanding and computing are connected to symbolic notation and procedures. Procedural knowledge develops meaning for symbol, recalling procedures and using procedures. Conceptual knowledge provides students symbolic language to express ideas, solve tasks involving routines or procedures, and promote the use of new procedures. When students can't connect these types of knowledge together, they either "can't solve a problem or they may generate answers but not understand what they are doing."

"***Reasoning*** is the glue that holds mathematics together." NRC explains reasoning as explaining and justifying a solution to a problem. Hiebert would describe reasoning as an important part of the classroom's social culture. Students need to communicate through talking, writing, listening, or seeing. For example, it is valuable for students to reason why multiplying a number greater than zero to a number less then zero will result in a number less than zero. By offering opportunities for communication among students, not only will they learn from their peers, self-correct misconceptions, they will also learn to be confident in their thinking.

Engagement addresses the involvement of students in the personal commitment to making sense of mathematics. This means students should not see math as a set of procedures or rules, but as a subject that logically makes sense, has meaning, and allows them to figure out a problem. Engagement is the motivation that students need to see themselves as confident and competent mathematical learners.

It has been found that higher mathematics scores are related to adequate access to computer technology (Wenglinsky, 1998). Additionally, computer use that utilized simulations, modeling, and applications kept students engaged in learning, improving their proficiency in math.

Aha!Math engages the learner through four components:

Lessons introduce and explain mathematical topics or concepts with the assistance of a "digital coach," engaging animation, and auditory and visual cueing.

Printable Activities reinforce the lesson and, depending on the applied learning community, stimulate further math exploration, inquiry, and discovery.

Practices give the student an opportunity to apply what he or she has learned in a highly motivating, game-like atmosphere. Practice activities, utilizing specially designed interactive programming, keep the student engaged for hours in non-repetitive challenges.

Quizzes illustrate student progress and understanding for the student and the teacher. Each quiz is electronically scored and reported.

Mathematics learning time must extend beyond a short period of daily instruction, affording time to apply, connect, and explore.

The National Research Council reports in *Helping Children Learn Mathematics* that the amount of time devoted to teach mathematics is too short. This is compounded by textbooks that are packed with topics (trying to address all the learning expectations states have in standards documents) and an instructional approach that is shallow and repetitive. Key ideas or big concepts are often hidden and difficult to identify. Teachers can actually spend too much of the little time devoted to math on skills that are not connected to important concepts or that do not extend deep enough to allow the learner to make appropriate connections.

A state-by-state review of recommended instructional time highlights the fact that most states prescribe approximately one-third the amount of instructional time for elementary mathematics as they do for elementary reading/language arts. Averages show that students spend between 700 and 750 minutes per week engaged in reading/language arts instruction compared with 200 to 250 minutes engaged in math instruction. Reading/language arts activities often also extended into other content areas such as social studies and science, leaving math lagging again.

NRC reports math achievement increases with time and opportunity. It recommends a minimum of one hour of math instruction each day, that instruction should be around all strands of math, and that concepts should be developed in a focused curriculum both within and across grade levels. Thus, Aha!Math's design specifically creates opportunities for students to extend their mathematics learning time during school hours, before or after school, at home, or as part of their work in programs such as after-school programs, independent study or summer school.

The lessons and activities in Aha!Math are intended to increase the quantity and quality of learning time. Aha!Math enables students to work independently, in a small group, or with the whole class. Because Aha!Math is Web-delivered, students may also engage in its lessons at anywhere they have a computer and Internet access, for example, at home with a family member. Applying, exploring, and making connections together create an entirely new way to ensure students have much more learning time.

Supplementing classroom instruction with technology solutions that include learning communities enhances the quality of math instruction and yields higher levels of achievement.

Technology is becoming a common classroom tool. As with all common tools, most are widely used, some with greater efficiency than others, and some inappropriately. The software or programming developed to support instruction can take a student to new levels of visualizing, understanding, and application. Yet, in the real world, the world for which we are preparing our students, technology is not a stand-alone tool. To be successful, what one gains from technology must be integrated with other skills, knowledge, and abilities in a variety of contexts to genuinely achieve higher levels of learning.

Analyzing, reasoning, creating, and thinking beyond, can be stimulated with good technology experiences. However, this synergistic field of boundless opportunity relies heavily on communicating with others. Thus, technology best supports learning in inquiry-based environments in which students are engaged in exploring, discovering, and creating knowledge in community settings (Vygotsky, 1978; Rogoff, 1991).

Aha!Math encourages the formation of a community of learners for each student. While each student's community may differ, the goal is to ensure the student has further developmental opportunities. Learning communities can be small or large, occur during or beyond the school day, and vary in duration. Aha!Math begins each student's community of learners with a digital coach, an animated character that offers positive feedback and guides the student. The classroom teacher may extend the student's communities to include, for example, a "buddy" in class, the whole class, after-school mentors, parents, small peer groups, and cross-age tutors.

Additionally, Aha!Math recognizes that mathematics is used in multiple communities, for example, in fields like science, technology, social sciences and video games, each with its own way of learning, thinking, creating and communicating. The context that each of these communities has with mathematics is different. Aha!Math simulates the experience of the community and the six ways of learning necessary to connect with multiple learning communities (Mayer, 2003):

- Essential vocabulary
- Understanding of the core concepts
- Competencies for application of concepts
- Problem-solving strategies
- Ways of creative thinking
- Process of communication of data and new understanding

How does a student learn to fully participate and communicate in a community of learners? Sometimes sharing ideas, asking questions, and letting other know what you don't know can be intimidating and embarrassing. The easiest way to introduce a child to the world of learning through community sharing and inquiry is to give him or her a community with the privacy necessary to become comfortable with thinking through what he or she knows and doesn't know. Aha!Math's Web-based program begins this process with the student, the computer, and the digital coach, who gives the student immediate feedback. These animated, supportive characters serve as models for positive interaction around learning.

A key element to improved academic achievement in math is the striking importance of collaboration (Follansbee, 1997). Classroom teachers already struggle to get enough time for direct instruction with practice; collaboration and creating learning communities takes additional time. Aha!Math builds in interaction for the student with his or her digital coach, and creates multiple opportunities for the teacher to involve the student in additional learning communities. It simulates a community of learners with its digital coaches and then that plays itself out in classroom. Aha!Math and the community of learners instructional strategy can expand learning time and content. Mathematics can be applied to other subject areas, such as science, economics, and graphic arts. And, mathematics can begin to permeate the student's world, giving further relevance to mathematics.

Innovative and creative technology solutions motivate students to learn.

Motivating students to learn mathematics is a multidimensional process involving many different factors. First, we must consider what compels a student to learn. Second, we must pay attention to each student's evaluation of his or her learning success. And, third, we must incorporate the strategies known to engage, challenge, and prompt further learning.

Motivation to learn comes from several sources. To begin, motivation is learned (Middleton, Spanias, 1999). Students start school with limited connections between effort and ability. Soon it becomes evident that those who excel in math master the assigned task in a timely manner, striving to move onto the next task. Thus, in part, motivation to learn is greatly enhanced by a student's ability to maintain a pace that holds his or her interest, is not lacking in challenge, and, yet, supports the development of confidence along the way. For this reason Aha!Math's content is not timed, but rather allows students to progress at their own pace.

The motivation to learn varies. One student might want to learn because he sees value in the topic or e finds it enjoyable. Another student might engage in learning for the grade, stickers, or verbal praise. While both intrinsic and extrinsic reasons can motivate learning, intrinsic motivation contributes to developing higher levels of confidence, greater enjoyment in mathematics, and the development of perseverance, while extrinsic motivation can often break down. Good grades, special assemblies, or certificates, and conversely, poor grades, lost recess, or retention "…can bring students to believe that learning activities are not worth doing in their own right…." (Stiggins, 2001).

Students' lack of early success in mathematics also plays a part in their motivation to learn. Students are willing to participant when they know where they are heading (Stiggins). As students engage in tasks that are moderately difficult for them, they feel successful and develop self confidence. As they have some success, they begin to take risks in learning. Intrinsic motivation occurs and students find themselves enjoying the work and thus, working harder. Teachers can be pleasantly surprised when a student asks to have additional problems for homework or that he or she is willing to give up recess to work on mathematics.

To support students to early success, and thus to feel motivated, Aha!Math's Web-delivery allows for immediate and multiple forms of feedback. The design of Aha!Math is interactive and dynamic, which creates a "learn by doing" environment. Additionally, as students progress through lessons, they receive feedback, can refine their thinking and understanding, and build new knowledge. The practice components and the feedback received through the practices, provide learners with a sense of autonomy needed to further add to intrinsic motivation. Descriptive feedback such as progress toward the learning target permeates the lessons. Evaluative feedback is also part of the program. Students and teachers can see and access their assignments in one place and can see their progress, too. These components are what students need to experience to develop their sense of enjoyment and increase their confidence, and thus, their motivation to learn.

The next factor affecting student motivation to learn math, widely exposed yet often ignored, encompasses the performance of groups like girls and minorities. Girls, for example, have been socialized that math is not their forte. Several studies report findings that girls more than boys accredit their lack of success in mathematics to their lack of ability and/or lack of confidence. This is perpetuated when teachers tend to call on and interact more with boys during math instruction. To counter these challenges, Aha!Math uses digital coaches that represent girls and underperforming subgroups. Because feedback is based on performance, beliefs about a particular learner do not impact the amount or type of feedback a learner receives through Aha!Math.

Furthermore, scripts and animations allow students to access information through multiple modalities that support the various types of learners. The lessons themselves present concepts in multiple formats. For example, in learning multiplication facts, students engage in learning about factors and their relationships in the multiplication table. This concept is also connected to properties of numbers and how to get the same value using the same numbers in a different order, and for some products, different numbers result in the same value. Aha!Math also supports conceptual understanding by visually displaying a concept, then using animation to “morph” the concept into a computation, so students literally see the computation.

We must begin to use knowledge that influences motivation as we design instructional programs to improve student learning outcomes in mathematics. As Stiggins clearly states, “… the only students who learn are those who want to learn” (Stiggins, 2001). Instructional practices can influence motivation so students can not only do the math but also learn to enjoy the math. Learning.com understands the importance of motivating student learning and, hence, is using technology as a critical design component.

Aha!Math’s Solution	Research Reference
Aha!Math is based on NCTM’s Curriculum Focal Points for grades K-5	Curriculum design is as important as technology design: (MacArthur, Graham, Schwartz & Schafer, 1995; Thomas & Hoffmeister, 1996.
Aha!Math creates additional opportunities for increased types and quality of visual and auditory cueing and coaching to enhance mathematics instruction	Simulations are effective in improving student achievement (Hennessy et al., 1995; Akpan & Andre,2000; Vahey, Enyedy, & Gifford, 2000.
Printable Actions or Collaborative Activities extend technology-based learning	Classroom in which technology is used should include collaborative opportunities for students to engage in higher levels of thinking, inquiry and exploration. (Bruce & Levin, 1997)
Aha!Math’s Web-delivered Practice Activities and Quizzes provide timely	Timely, rich, and immediate feedback improves student learning and motivation.

feedback for students and teachers.	(Wong, 2001)

Learning.com also created Aha!Math to contain features that allow it to best employ its dynamic interactivity. These include:

- Transformation or manipulation within a particular notation system. For example, adding two numbers with regrouping.
- Translations between notation systems. For example, adding two numbers and showing the sum as both with symbols and on a number line.
- Creating and testing mathematical models. For example, simulation software can be used to make several triangles and measure all three angles, testing for the sum of three angles.
- Links between relationships and the processes or concepts.

Additionally, Aha!Math's features include:

- Dynamic media versus static media. For example, if addressing rotations, the computer animation of the shape will allow the shape to rotate. If regrouping numbers, the student can see a visual animation of bundles of 10 being made.
- Interactive. Digital manipulatives are an example of interactive materials. Interactive use of technology allows for linked representation of materials, which is key to building connections of ideas and actions and internalizing a concept. For example, seeing and using a calculator, a number line, and manipulatives to add two numbers.

Quality teaching and coaching improves math achievement.

Teaching mathematics with understanding requires having teachers with solid knowledge. While many studies focused on teacher knowledge, Shulman (Shulman 1996) analyzed in new ways the knowledge needed to effectively teach. Many researchers (Borko & Putnam, 1996; Fennema & Franke, 1992; Grossman, 1991; Leinhardt & Smith, 1985) expanded upon Shulman's ideas by investigating how knowledge used in teaching differs from knowledge used within a pure discipline. Whereas each study on teacher knowledge examined similar details of teacher understanding, each researcher categorized the details using differing specificity and labels. To describe how Aha!Math helps build teacher knowledge, this paper will use the broad categories of subject matter knowledge and beliefs, general pedagogy and beliefs, and pedagogical content knowledge and beliefs (Borko and Putnam, 1996).

Subject matter knowledge. Subject matter knowledge is more than the knowledge used to pass a content-specific test. Understanding mathematics requires knowing the key concepts, themes and relationships (California Department of Education, 1997; National Council of Teachers of Mathematics, 2000), and recognizing the building blocks or simpler ideas that lead into a concept (Ma, 1999). It is the knowledge of specific content, themes or relationships in the subject (Leinhardt and Smith, 1985). It means knowing the particular ideas and principles that guide the subject. Subject matter knowledge includes representing and explaining concepts, inquiring about the subject matter, and demonstrating conceptual understanding (Mosenthal and Ball, 1992). Subject matter knowledge also involves understanding how new knowledge is created and how knowledge is connected or related to other knowledge within the discipline (Shulman, 1986).

Aha!Math relates subject matter knowledge and critical concepts by illustrating through multiple representations. For example, Aha!Math shows the relationship between a math concept and a math procedure. Lessons are built around the connections between conceptual understanding and procedural fluency. Notes for teachers provide valuable links, extensions of learning and anticipate possible student questions. Its focus on NCTM focal points also serves as a model for teachers to use as they go beyond this supplemental curriculum to teach other key mathematical concepts.

General pedagogical knowledge. This knowledge is content-neutral and refers to the knowledge and beliefs teachers have with regard to curriculum, learning and the learner. A teacher demonstrates his or her general pedagogical knowledge when he or she plans lessons for student understanding and incorporates goals and objectives within a curriculum or within a lesson, and is further substantiated through the teacher's ability to modify plans, sequence instruction, and provide multiple forms of assessment (Darling-Hammond et al., 1999). General pedagogical knowledge supports an environment that allows for reflection, collaboration, feedback and communication. It is a teacher's responsibility to know what students learn, how they learn, and where they learn in order for powerful teaching to take place (Brandt, 1998).

Additionally, it requires the knowledge to handle issues of student motivation and behavior. Utilizing these compelling teaching conditions provides robust experiences for students to learn with understanding.

Aha!Math builds several components of general pedagogical knowledge into its structure. It incorporates effective ways to signal the beginning and end of the various segments within a lesson. For example, providing an engaging and interactive environment for learning not only motivates students to learn but also distracts students from choosing unacceptable classroom behaviors. Aha!Math also gives critical attention to feedback. Students receive feedback as they move through lessons that lead toward proficiency of the learning target. Aha!Math keeps in mind the importance of reflection, collaboration, and communication. For example, prompts during the learning ask students to reflect on important aspects of the learning. Sometimes these ideas are worked on in collaboration with others.

Pedagogical content knowledge. The blending of content and pedagogy defines the third domain of teacher knowledge. Pedagogical content knowledge connects content and pedagogy. It is having the knowledge to represent a topic in multiple ways geared specifically toward learners (Borko & Putnam, 1996). It means understanding and recognizing the common conceptions and misconceptions of a topic (Fennema & Franke, 1992). Pedagogical content knowledge includes understanding the representations of the topic and the learner well enough to use appropriate curriculum and learning tools for learners to construct meaning.
The content of Aha!Math is represented to students in multiple ways, providing teachers with multiple ways to present information based on the learner's needs. For example, animations introduce concepts and procedures, guided practices allow them to continually apply their knowledge of the concept in yet another way. Manipulatives and learning tools, such as games and simulations, are specifically chosen to match each concept and how students learn the concept. Furthermore, typical misconceptions are addressed through posed questions, error analysis, and guided practice exercises.

Because Aha!Math is available to students to use anywhere there is a computer and the Internet, at home and throughout a community, for example, additional people beyond the teacher can coach or mentor the learner and, thus, extend instructional time.

Improving teacher knowledge through improving subject matter knowledge, general pedagogical knowledge, and pedagogical content knowledge is a pathway for changing mathematical understanding for both teachers and students (Stigler & Hiebert, 1999), and calls for teachers to be learners. Transforming the classroom demands "continuing efforts [of teachers] to learn and improve" (National Council of Teachers of Mathematics, 1991, p. 19). "Teaching for understanding requires teachers to be learners, constantly reviewing their practice and renewing their content and pedagogical knowledge" (McLaughlin & Talbert, 1993, p. 5). Aha!Math is a tool that teachers can use to build their knowledge. It models for teachers the critical connections in mathematical topics that they can extend to additional topics. As teachers continue to learn through their use of Aha!Math with students, the knowledge they need to be effective teacher will increase, giving them what they need to teach mathematics with understanding.

IV. CONCLUSION

Aha!Math was created to support teachers to improve student learning for grades K-5. By focusing on a much deeper, inquiry-based approach to learning math concepts and procedures, the Web-delivered design of Aha!Math fosters the development of foundational math skills. This foundation fortifies students with the skills, knowledge, and abilities to excel in mathematics, science, and other related fields.

Aha!Math is for educators who are concerned about:

- student achievement in mathematics,
- teachers who need support to become proficient and confident math instructors,
- limited resources such as instructional time, tutors, and hardware,
- the depth of mathematics instruction,
- the need for research-based instructional strategies (what works),
- student motivation to learn,
- the lack of parent involvement in learning,
- and, simply stated, students who aren't excited about mathematics.

The research about how students learn and how teachers most effectively teach has informed both the design and delivery of Aha!Math, creating a balanced approach to mathematics for K-5 students.

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VII. ABOUT THE AUTHOR

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Barclay Burns co-founded Learning.com with William Kelly in 1999. He received his doctorate in instructional psychology and technology from Brigham Young University. He has seven years' experience teaching in a public high school where he founded the "Community Bound" integrated education program. This program exposed high school students to real-world applications of their learning and successfully solicited corporate sponsorship of innovative programs and trips, such as two trips to Romania to paint orphanages. He has also worked on the development of BYU's extensive online distance learning applications for high school students, which includes the delivery of curriculum and grades to students and parents over the Internet. He was most recently in a post-doctoral program in Internet Strategy at the University of Cambridge in England where he taught and researched at the Judge Institute of Management.

